Journal of Engineering Physics and Thermophysics, Vol. 79, No. 1, 2006

GAS FLOW THROUGH A POROUS HEAT-RELEASING MEDIUM WITH ALLOWANCE FOR THE TEMPERATURE DEPENDENCE OF GAS VISCOSITY

V. A. Levin and N. A. Lutsenko

UDC 532.546

The gas flow through a porous heat-releasing medium is considered. It has been noted that account for the temperature dependence of the gas viscosity strongly influences the solution: the gas flow rate markedly decreases and a stronger heating occurs. Analysis of the flow of a gas with a temperature-dependent viscosity by the Sutherland formula has revealed two steady-state cooling regimes — stable and unstable ones. It has been shown that the possibility of the steady-state regime is determined not only by the problem parameters but by the initial conditions as well. The transient process from the state of rest in the absence of heat release to the state of the regime of induced filtration upon instantaneous switching-on of heat input is described.

Problem Formulation. In simulating the gas flow through a porous heat-releasing medium, the gas viscosity is usually assumed to be constant [1–4]. The present paper considers flows with a temperature-dependent gas viscosity by the Sutherland formula. The investigated gas flow through a solid, porous, homogeneous, stationary medium in which heat release occurs can arise under cooling of fuel elements. A similar model was proposed in [1] for describing the process of cooling the exploded unit of the Chernobyl NPP.

Suppose we have a fuel element of height H and a cold gas is conveyed under pressure into its lower part; the gas flows up through a porous medium, is heated as a result of the heat exchange, and flows out into a free space with a given pressure. The model is constructed on the assumption of two interacting continua [5]. Let us assume that the heat release in the solid phase is directly proportional to the reactant concentration, whose decrease rate is directly proportional to the concentration itself, and the volume and mass of the condensed phase change insignificantly and these changes can be neglected. We assume that the intensity of the interphase heat exchange is proportional to the phase-temperature difference at the considered point of the medium, for the gas the equation of state of a perfect gas holds, and the condensed phase is stationary and uniform. The system of equations describing such a process is of the following form:

$$(1-a)\,\rho_c c_c \frac{\partial T}{\partial t} = -\alpha\,(T - T_g) + Q_0\,(1-a)\,C + (1-\alpha)\,\lambda\,\frac{\partial^2 T}{\partial x^2}\,,$$

$$a\rho c_p \left(\frac{\partial T_g}{\partial t} + v_g \frac{\partial T_g}{\partial x}\right) = \alpha\,(T - T_g) + a\left(\frac{\partial p}{\partial t} + v_g \frac{\partial p}{\partial x}\right) + a^2 \frac{\mu}{k_1} v_g^2 + a\frac{\partial}{\partial x}\left(\lambda_g \frac{\partial T_g}{\partial x}\right),$$

$$\rho\,(1 + \chi_m\,(1-a))\left(\frac{\partial v_g}{\partial t} + v_g \frac{\partial v_g}{\partial x}\right) = -\frac{\partial p}{\partial x} - \rho g - a\frac{\mu}{k_1} v_g\,, \quad \frac{\partial a\rho}{\partial t} + \frac{\partial a\rho v_g}{\partial x} = 0\,, \qquad (1)$$

$$p = \rho R T_g\,, \quad \frac{\partial C}{\partial t} = -k_2 C\,.$$

Institute of Automatics and Control Processes, Far-East Branch of the Russian Academy of Sciences, 5 Radio Str., Vladivostok, 690041, Russia; email: nickl@inbox.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 79, No. 1, pp. 35–40, January–February, 2006. Original article submitted June 11, 2004; revision submitted May 5, 2005.

In investigating the nonisothermal filtration of a liquid, the dynamic viscosity, as a rule, is assumed to be temperature-dependent [6]; however, in simulating the nonisothermal filtration of a gas, the viscosity is often assumed to be constant. Further we will consider that the dynamic gas viscosity depends on temperature according to the Sutherland formula, and the heat conductivity of the gas is proportional to its viscosity:

$$\mu = c_{s1} \frac{T_g^{1.5}}{c_{s2} + T_g}, \quad \lambda_g = \frac{c_p}{\mathrm{Pr}} \mu .$$

Let us introduce the following designation: $u = av_g$. From the last equation of system (1), find the expression for the reactant concentration: $C = \exp(-k_2 t)$. The last term in the second equation of system (1) can be neglected, since the heat conductivity of the gas is low. Thus, the system of equations describing the gas flow through the porous heat-releasing medium will take on the following form:

$$(1-a)\rho_c c_c \frac{\partial T}{\partial t} = -\alpha (T - T_g) + Q_0 (1-a) \exp(-k_2 t) + (1-a)\lambda \frac{\partial^2 T}{\partial x^2},$$

$$\rho c_p \left(a \frac{\partial T_g}{\partial t} + u \frac{\partial T_g}{\partial x} \right) = \alpha (T - T_g) + \left(a \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} \right) + \frac{c_{s1}}{k_1} \frac{T_g^{1.5}}{c_{s2} + T_g} u^2, \tag{2}$$

$$\frac{p(1 + \chi_m (1-a))}{a^2} \left(a \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) = -\frac{\partial p}{\partial x} - \rho g - \frac{c_{s1}}{k_1} \frac{T_g^{1.5}}{c_{s2} + T_g} u,$$

$$a \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0, \quad p = \rho R T_g .$$

At the inlet into the porous element, the gas temperature and pressure are known, and at the outlet — the pressure is known, since the gas flows out into an open space. The conditions for the heat exchange at the inlet and outlet from the porous element are also known. Thus, the boundary conditions for system (2) have the form

$$p\big|_{x=0} = p_0(t), \quad T_g\big|_{x=0} = T_{g0}(t), \quad \lambda \frac{\partial T}{\partial x}\bigg|_{x=0} = a_* \left(T\big|_{x=0} - T_{g0}\right),$$

$$p\big|_{x=H} = p_h, \quad \frac{\partial T_g}{\partial x}\bigg|_{x=H} = 0, \quad \lambda \frac{\partial T}{\partial x}\bigg|_{x=H} = a_* \left(T_g\big|_{x=H} - T\big|_{x=H}\right). \tag{3}$$

To solve system (2), it is also necessary to give the values of the sought quantities at the initial instant of time.

Consider now the steady-state and time-invariable process. In this case, the heat release in the solid phase is constant. The inertial terms in the momentum conservation law can be neglected. Then the third equation of system (2) transforms into the Darcy law. By simple manipulations, we get the system of equations describing the stationary gas flow through the porous heat-releasing medium:

$$\alpha (T - T_g) = Q_0 (1-a) + (1-a)\lambda \frac{d^2 T}{dx^2}, \quad \rho u c_p \frac{dT_g}{dx} = \alpha (T - T_g) - \rho g u,$$

$$\frac{dp}{dx} = -\rho g - \frac{c_{s1}}{k_1} \frac{T_g^{1.5}}{c_{s2} + T_g} u, \quad \frac{d\rho u}{dx} = 0, \quad p = \rho R T_g . \tag{4}$$

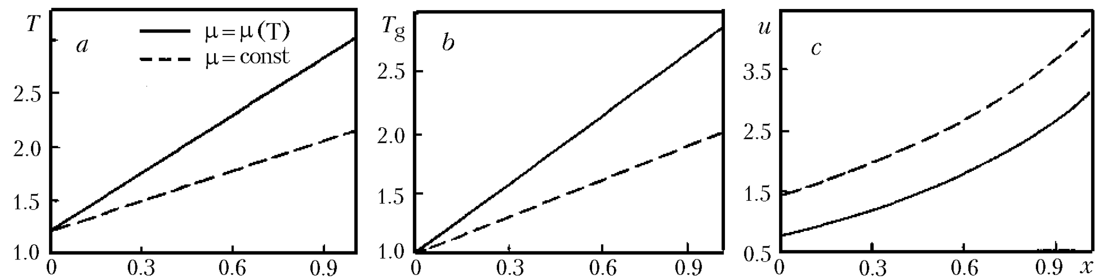


Fig. 1. Distributions of the condensed-phase temperature (a), the gas temperature (b), and the gas filtration rate (c) over the element height.

Note that if the heat release in the solid phase is substantial, then the last term in the second equation of system (4) can be neglected. Let us write the boundary conditions for system (4):

$$p\big|_{x=0} = p_0 \,, \quad T_{\mathrm{g}}\big|_{x=0} = T_{\mathrm{g}0} \,, \quad p\big|_{x=H} = p_{\mathrm{h}} \,, \quad \lambda \frac{dT}{dx}\bigg|_{x=0} = a_* \, (T\big|_{x=0} - T_{\mathrm{g}0}) \,, \quad \frac{dT}{dx}\bigg|_{x=H} = 0 \,. \tag{5}$$

Systems (2) and (4) are solved in the dedimensionalized form, but the dimensionless variables thereby are input in the following way: $\tilde{x} = x/H$, $\tilde{t} = t/t_*$, and $\tilde{u} = u/u_*$, where t_* and u_* are the characteristic values of the time and filtration rate of the gas; $\tilde{p} = p/p_*$, $\tilde{\rho} = \rho/\rho_*$, $\tilde{T} = T/T_*$, and $\tilde{T}_{\mathrm{g}} = T_{\mathrm{g}}/T_*$; here p_*, ρ_*, and T_* are the gas pressure, density, and temperature under "normal" conditions.

Influence of the Temperature Dependence of the Gas Viscosity on the Solution. Further, unless otherwise specified, we shall consider systems (2) and (4) with the following parameters:

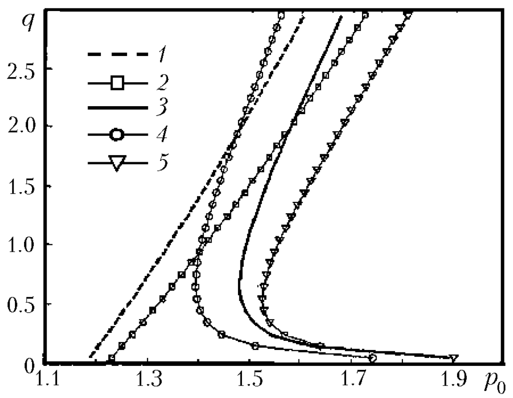
$$\begin{gathered}
H = 10 \text{ m} \,, \quad t_* = 1 \text{ sec} \,, \quad u_* = 1 \text{ m/sec} \,, \quad T_* = 300 \text{ K} \,, \quad p_* = 10^5 \text{ Pa} \,, \quad \rho_* = 1.2 \text{ kg/m}^3 \,, \\
\rho_{\mathrm{c}} = 2.2 \cdot 10^3 \text{ kg/m}^3 \,, \quad c_{\mathrm{c}} = 9.2 \cdot 10^2 \text{ J/(kg·K)} \,, \quad a = 0.3 \,, \quad \alpha = 10^3 \text{ J/(m}^3\text{·K·sec)} \,, \\
c_p = 10^3 \text{ J/(kg·K)} \,, \quad c_{\mathrm{s}1} = 1.458 \cdot 10^{-6} \text{ kg/(m·sec·}\sqrt{\text{K}}) \,, \quad c_{\mathrm{s}2} = 110.4 \text{ K} \,, \\
k_1 = 10^{-8} \text{ m}^2 \,, \quad Q_0 = 10^5 \text{ J/(m}^3\text{·sec)} \,, \quad \lambda = 1.2 \text{ J/(m·K·sec)} \,, \\
k_2 = 10^{-7} \text{ 1/sec} \,, \quad \chi_{\mathrm{m}} = 0.5 \,, \quad a_* = 10 \text{ J/(m}^2\text{·K·sec)} \,.
\end{gathered} \tag{6}$$

Solve system (4) with the boundary conditions obtained from (5) for the following values of the dimensionless quantities:

$$p_0 = 1.5 \,, \quad T_{\mathrm{g}0} = 1 \,, \quad p_{\mathrm{h}} = 1 \,. \tag{7}$$

The system is solved as in [4]. Figure 1 shows the distributions of the condensed-phase temperature, the gas temperature, and the gas filtration rate over the element height. For comparison, the same figure gives the above distributions for the case of a constant dynamic viscosity of the gas [4] at $\mu = 2 \cdot 10^{-5}$ kg/(m·sec). The gas flow rate is equal to 1.1167 in the first case and 2.1465 in the second case. Since the gas viscosity is temperature-dependent, its mean value turns out to be higher and, therefore, the total friction force increases, leading to a decrease in the gas velocity, a marked decrease in the flow rate, and a stronger heating.

Figure 2 shows the dependences of the gas flow rate on its pressure at the inlet into the heat-releasing element for the case of both variable gas viscosity (with various values of constants in the Sutherland formula) and constant gas viscosity (with various values of viscosity). Computations have been performed for the boundary conditions obtained from (5) for the following values of the dimensionless quantities:



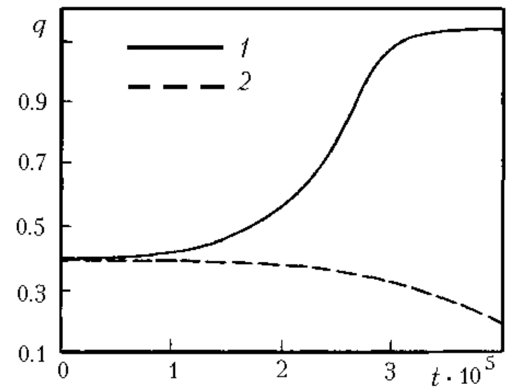


Fig. 2. Flow rate of the gas versus its pressure at the inlet into the element: 1) $\mu = 2 \cdot 10^{-5}$; 2) $2.5 \cdot 10^{-5}$ kg/(m·sec); 3) $\mu = \mu(T)$, $c_{s1} = 1.458 \cdot 10^{-6}$ kg/(m·sec$\sqrt{K}$); c_{s2}, 110.4 K (corresponds to parameters (6)); 4) $\mu = \mu(T)$, $c_{s1} = 1.15 \cdot 10^{-6}$ kg/(m·sec$\sqrt{K}$); c_{s2}, 110.4 K; 5) $\mu = \mu(T)$, $c_{s1} = 1.458 \cdot 10^{-6}$ kg/(m·sec$\sqrt{K}$); c_{s2}, 10 K.

Fig. 3. Change in the gas flow rate on the system going from the unstable regime of cooling to: 1) the steady-state regime; 2) indefinite heating.

$$T_{g0} = 1 \,, \quad p_h = 1 \,. \tag{8}$$

As is seen from Fig. 2, with allowance for the temperature dependence of the gas viscosity, the critical value of the gas pressure at the inlet into the heat-releasing element, below which the steady-state cooling regime does not exist, is attained at a zero gas flow rate, unlike the flow of a constant-viscosity gas. From Fig. 2 it is also seen that in the flow of a gas with a temperature-dependent viscosity two steady-state regimes of cooling exist: to one and the same value of the gas pressure at the inlet into the element two values of the gas flow rate can correspond. To analyze the stability of these regimes, system (2) with boundary conditions (3) was solved by the finite-difference method [7] under the condition of constant pressure and temperature of the gas at the inlet into the element. It turned out that the cooling regime corresponding to the lower flow rate of the gas at its given pressure at the inlet into the element is unstable, whereas the steady-state regime corresponding to the higher flow rate of the gas at its given pressure at the inlet into the element proved to be stable. From the unstable steady-state cooling regime the system either slowly goes to the stable steady-state regime or is heated indefinitely. This is graphically demonstrated by Fig. 3, showing the change in the gas flow rate with time. The choice of the system behavior in the unstable steady-state regime depends on the slightest change in the gas pressure at the inlet into the element: if the pressure decreases insignificantly, then indefinite heating occurs, and if it increases slightly, then the system goes to the stable steady-state regime. In nature, pressure oscillation is caused by natural fluctuations of all quantities, and in numerical calculations they arise from the ever-present calculation error.

The transition to the regime of indefinite heating at a gas pressure at the inlet above the critical value can be explained as follows. At a fairly low gas flow rate, a considerable heating of the element in its upper part occurs. Because of the temperature dependence of viscosity, it can reach a value in the strongly heated region so large that the increased friction force will markedly impede the gas flow and its rate will continue to decrease and, as a result, the element will continue to be heated.

Thus, it has been shown that in simulating the gas flow through a porous heat-releasing medium, it is necessary to take into account the temperature dependence of the gas viscosity.

Problem on Switching Induced-on Filtration at the Onset of Heat Release. Consider now the following problem. Heat release in the solid phase before the initial instant of time is absent and the pressure at the inlet into the element and at its outlet corresponds to atmospheric pressure at given heights; consequently, air motion in the element is absent. At the initial instant of time, heat release in the solid phase begins and simultaneously the gas pressure at the inlet into the element increases.

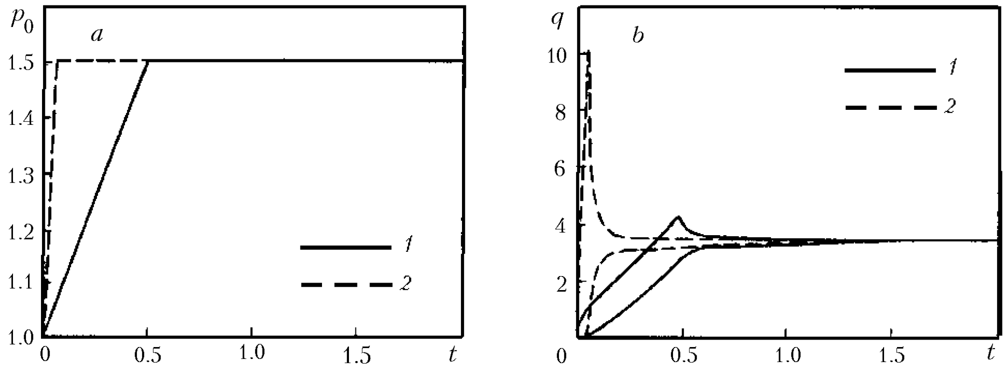


Fig. 4. Gas pressure at the inlet into the element (a) and gas flow rate at the inlet into the element (upper branches of the graph) and at the outlet from the element (lower branches of the graph) (b): 1) $B = 1$; 2) 10.

Let the problem parameters be defined according to (6). The pressure at the inlet into the element increases linearly to 1.5 and then remains constant. Then the boundary conditions are obtained from (3) at the following values:

$$p_0 = \exp\left(gH/(RT_*T_{g0})\right) + tB\,, \quad \text{if} \quad \exp\left(gH/(RT_*T_{g0})\right) + tB < 1.5\,;$$

$$p_0 = 1.5\,, \quad \text{if} \quad \exp\left(gH/(RT_*T_{g0})\right) + tB \geq 1.5\,; \tag{9}$$

$$T_{g0} = 1\,; \quad p_h = 1\,.$$

System (2) is solved by the finite-difference method as in [7]. In this case, transition to the steady-state regime of cooling occurs, since at the given problem parameters and boundary conditions the steady-state regime of cooling exists, as is seen from Fig. 2. And though the new regime will not be completely steady-state, since the heat release in the solid phase slowly decreases, this decrease can often be neglected.

The transition to the steady state can be split into two stages. At the first stage there is a rapid and drastic change in the gas pressure, density, and rates of filtration and flow. This change markedly slows down shortly after the gas pressure at the inlet into the element settles at a constant value and the second stage of the transition to the steady state begins: there occurs a slow heating of the element, causing a slow change in all the other quantities sought. The new stationary solution is the asymptotics to which the system solution tends.

Consider in more detail the first stage of the transition to the steady state. Figure 4 shows the gas pressure at the inlet into the element and the gas flow rate for two variants of the problem under considerations differing in the rapidity of pressure increase at the initial instant of time. The upper branches of the gas flow rate graphs correspond to the values at the inlet into the element, and the lower ones correspond to the outlet values. As is seen from Fig. 4, the inlet and outlet gas flow rates approximately equalize fairly rapidly — this instant of time is the termination of the first stage of the transition to the steady state. The graph of the gas flow rate at the inlet into the element has a clearly defined peak depending on the rapidity of the gas-pressure increase at the inlet into the element, which is noticeable even at a rather slow gas pressure increase at the inlet.

Consider this problem now with other boundary conditions. Let the inlet pressure increase linearly to 1.4 and upon reaching this value remain constant, the other boundary conditions being left without a change. Then the boundary conditions are obtained from (3) at the following values:

$$p_0 = \exp\left(gH/(RT_*T_{g0})\right) + tB\,, \quad \text{if} \quad \exp\left(gH/(RT_*T_{g0})\right) + tB < 1.4\,;$$

$$p_0 = 1.4\,, \quad \text{if} \quad \exp\left(gH/(RT_*T_{g0})\right) + tB \geq 1.4\,; \tag{10}$$

$$T_{g0} = 1\,; \quad p_h = 1\,.$$

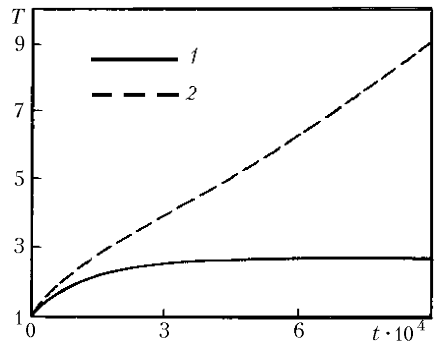


Fig. 5. Change in the condensed-phase temperature at the outlet from the element in the cases of: 1) transition to the steady state; 2) indefinite heating.

In this case, as is seen from Fig. 2, the steady-state regime of cooling does not exist. The system solution (2) confirms this: there occurs indefinite heating which is bound to terminate in solid-phase melting and disturbance of the above-described cooling process. However, this cooling process can also be split into two stages. The first stage is analogous to the above first stage of transition to the steady state: there is also a rapid and drastic change in the gas pressure, density, and rates of filtration and flow, which markedly slows down shortly after the inlet pressure settles at a steady state. The second stage of heating differs from the above stage in that element heating lasts indefinitely. Figure 5 shows the change in the solid-medium temperature at the outlet from the fuel element for both cases of the problem considered: transition to the steady state and indefinite heating.

Thus, if heat release in the solid phase begins simultaneously with the switching-on of air pumping at the inlet into the element, then no catastrophe occurs at the initial instant of time. Depending on the air pressure that has settled at a constant value at the inlet into the element, either the steady-state regime of cooling is realized or indefinite heating of the element leading to condensed-phase melting and disturbance of the cooling process occurs. Heating of the element to the melting temperature of the solid medium requires a rather long time, which will make it possible to increase the air pressure at the inlet into the element to a value at which the cooling regime will reach the steady state.

NOTATION

a, porosity; a_*, heat-transfer coefficient, J/(m^2·K·sec); B, parameter defining the rate of increase in the gas pressure at the inlet into the element at the initial instant of time; C, reactant concentration; c_c, heat capacity of the condensed phase, J/(kg·K); c_p, heat capacity of gas at a constant pressure, J/(kg·K); c_{s1}, constant in the Sutherland formula, kg/(m·sec$\sqrt{\text{K}}$); c_{s2}, constant in the Sutherland formula, K; g, gravitational acceleration, m/sec^2; H, fuel-element height, m; k_1, permeability coefficient of the condensed phase, m^2; k_2, coefficient defining the decrease in the heat release, 1/sec; p, gas pressure, Pa; Pr, Prandtl number; q, gas flow rate, kg/(m^2·sec); Q_0, constant defining the rate of heat release, J/(m^3·sec); R, gas constant, m^2/(sec^2·K); t, time, sec; T, condensed-phase temperature, K; T_g, gas temperature, K; u, rate of gas filtration, m/sec; x, Euler coordinate, m; v_g, gas velocity, m/sec; α, constant defining the interphase heat-exchange intensity, J/(m^3·K·sec); λ, heat conductivity of the condensed phase, J/(m·K·sec); λ_g, heat conductivity of gas, J/(m·K·sec); μ, dynamic viscosity of gas, kg/(m·sec); ρ_c, condensed-phase density, kg/m^3; ρ, gas density, kg/m^3; χ_m, factor of apparent mass taking into account the inertial interaction of phases in their accelerated relative motion. Subscripts: c, condensed phase; g, gas; h, outlet from the element; m, mass; 0, values at the inlet into the element.

REFERENCES

1. V. P. Maslov, V. P. Myasnikov, and V. G. Danilov, *Mathematical Simulation of the Exploded Unit of the Chernobyl NPP* [in Russian], Nauka, Moscow (1987).
2. V. P. Maslov, Effects of superheating in filtration media, *Dokl. Ross. Akad. Nauk*, **326**, No. 2, 246–250 (1992).

3. V. A. Levin and N. A. Lutsenko, Steady-state regime of filtration cooling of a porous heat-releasing element, in: *On the 30th Anniversary of the Institute of Automatics and Control Processes of the Far-East Branch of the Russian Academy of Sciences* [in Russian], Jubilee Collection of Papers, IAPU DVO RAN, Vladivostok (2001), pp. 151–159.
4. N. A. Lutsenko, One-dimensional steady-state regime of gas filtration through a layer of a stationary heat-releasing condensed material, *Dal'nevostochnyi Mat. Zh.*, **3**, No. 1, 123–130 (2002).
5. R. I. Nigmatulin, *Principles of the Mechanics of Heterogeneous Media* [in Russian], Nauka, Moscow (1978).
6. B. T. Zhumagulov and V. N. Monakhov, *Hydrodynamics of Oil Extraction* [in Russian], KazgosINTI, Almaty (2001).
7. N. A. Lutsenko, Nonstationary regimes of cooling a porous heat-releasing element, *Mat. Model.*, **17**, No. 3, 120–128 (2005).